Note on Discretization of a Laplace-Domain Low-Pass Filter

A first-order low-pass filter in the Laplace domain is defined by a transfer function with the following form:

$$\frac{Y(s)}{X(s)} = \frac{a}{s+a} \tag{1}$$

where *X(s)* and *Y(s)* are the input and output respectively and *a* is the filter pole or cutoff frequency.

Rearranging:

$$sY(s) + aY(s) = aX(s) \tag{2}$$

The *s* operator in the Laplace domain is equivalent to taking the derivative in the time domain. Eq. (2) can therefore be expressed in the time domain as:

$$\frac{dy}{dt} + ay(t) = ax(t) \tag{3}$$

Discretizing Eq. (3):

$$\frac{y_n - y_{n-1}}{T} + ay_n = ax_n \tag{4}$$

where y_n is the filter output of the current time step, y_{n-1} is the filter output of the previous time step, x_n is the current filter input and *T* is the sample period.

Rearranging (4) to solve for y_n yields the equation for the discrete-time, low-pass filter:

$$y_n - y_{n-1} + aTy_n = aTx_n$$

$$y_n(1 + aT) = y_{n-1} + aTx_n$$

$$y_n = \frac{y_{n-1} + aTx_n}{(1 + aT)} \tag{5}$$